# Morphology of the bronchial tree in man<sup>1</sup>

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HORSFIELD, KEITH, AND GORDON CUMMING. Morphology of the bronchial tree in man. J. Appl. Physiol. 24(3): 373-383. 1968.—The anatomical arrangements of the air passages in the human lung have been studied by preparing a cast with a thermosetting resin, followed by measurement of all the structures so delineated. The airways larger than 0.7 mm in diameter have been measured individually; those smaller have been measured by a sampling technique. The data obtained from all these measurements and some derived quantities are presented. A method for numbering the generations of branches counting up the bronchial tree is described.

bronchial anatomy; gas flow; morphometry

THE PART PLAYED by the structure of the lungs in determining their function has been the subject of recent work (4, 11, 13, 14). The interpretation of a variety of physiological tests is assisted by a knowledge of structure, and recent papers concerning the rate of gas mixing by diffusion serve to emphasize the importance of a knowledge of the geometrical arrangement of the air passages in the lung.

The transfer of a molecule of oxygen to its final destination in a biochemical reaction within a living cell is complex and, so far as the process of ventilation is concerned, involves two distinct processes—gas transport down the airways and gas mixing within them.

Gas transport is concerned with the characteristics of flow in branching tubes where mass movement is the dominant process. Gas mixing occurs in the terminal airways and here the physical process involved is gaseous diffusion so that understanding involves the application of physical knowledge to the biological situation. It is in these areas that quantitative measurements of the anatomy of the airways can be most helpful.

Other authors have undertaken this task of measurement (5, 12, 13). Ross (13) stressed the importance of asymmetry in the production of inequality of ventilation in the dog's lung and calculated the effect of such asymmetry in producing regional inequality of ventilation.

An analysis of the human bronchial tree has been made by Weibel (16). Using a resin cast of the larger airways he measured completely only as far as generation 5 and incompletely down to generation 10. The smaller structures were examined by conventional histological techniques.

The area between, involving the smaller bronchioles, was difficult and this he overcame by making the assumption of regular dichotomy and predicting therefrom the missing measurements. This approach has been valuable but suffers from the defect that inhomogeneity due to asymmetrical bronchial anatomy is automatically excluded.

The purpose of the study reported in this paper was to make measurements of the human bronchial tree as nearly complete as possible so that the functional effects of asymmetry could be studied. Before embarking on a description of the techniques of measurement and computation several points of principle should be discussed.

## PATTERNS OF BRANCHING IN BIOLOGICAL SYSTEMS

Branching systems may be organized in various patterns. At each node the parent branch may divide to produce two (dichotomy), three (trichotomy), or many (polychotomy) daughter branches. Another type of branching occurs when a lateral branch arises from a main stem, this being termed monopody. In dichotomy the growing point divides into two while in monopody a separate growing point arises on a branch which has been formed previously.

Apparent trichotomy may be found in a dichotomous system due to shortening of a branch to zero length.

#### Number of Branches

Symmetrical dichotomy. In this system each parent branch gives rise to two daughter branches of the same length and diameter. If the initial member of the system is designated as generation zero, then the total number of branches at any given generation is 2<sup>w</sup>, where w is the generation considered. Since this is an exponential statement, a plot of the logarithm of the number of branches against w will be linear.

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FIG. 1. Two methods of numbering the generations in a symmetrical dichotomously branching system. The branches may be numbered downward starting with the main stem as zero or upward starting with the end branches as zero.

The system may also be looked at in the opposite way, counting generations not from above down, but from below upward. Thus, the end branch is designated as generation zero and the number of branches in any generation u divisions up the system is given by  $2^{U-u}$ where U is the total number of generations. The comparison between the two techniques is shown in Fig. 1.

Asymmetrical dichotomy. Any · asymmetrical dichotomously branching system is one in which there is variation in the diameters or lengths of the branches in a given generation, or a variation in the number of divisions down to the end branches, or any combination of these. The following account applies to those in which there is a variation in the number of divisions down to the end branches. Such a system is shown in Fig. 2.

The laws of regular dichotomy are applicable to this system while branching continues. As soon as a branch terminates, however, any estimate of numbers of branches in a subsequent generation will be in error. Any given generation would, therefore, contain branches different in size and performing quantitatively different functions. Their grouping together is thus an unjustified restraint, as pointed out by Heiss (7) and von Hayek (6).

An approach to this difficulty is to consider that an asymmetrical system is a symmetrical one with a few bits missing, as indicated in Fig. 3. In the symmetrical system the generations are numbered upward, branches with similar structure and function (i.e., of the same generation) having the same number. When the asymmetrical system is derived from this by removal of some of the branches, the numbering of the remaining branches is unchanged so that branches similar in respect of structure and function still have the same number. In an asymmetrical biological system it is therefore more logical to number the branches upward than downward. This is done by numbering all the end branches 0, the next one up 1, the next 2, and so on. When two branches of different number meet, the sequence is continued from the higher number (Figs. 2 and 3).

## Radius of Branches

Symmetrical dichotomy. In most biological branching systems the radius of the daughter branches is less than that of the parent. If the proportionate diminution in each generation is the same, it is possible to formulate general laws about radii in a symmetrical system and these have been derived mathematically in the APPENDIX. Briefly, the radius  $r_w$  at any generation w is given by

$$r_w = K^w r_0$$

where K is a constant less than unity and  $r_0$  is the radius of the main stem. This applies when counting downward.

When counting upward, however, at generation u

$$\mathbf{r}_{\mathbf{u}} = \mathbf{K}_{1}^{\mathbf{u}} \mathbf{r}_{0}$$

when  $r_0$  is the radius of the end branch and  $K_1$  is a constant greater than 1.

Asymmetrical dichotomy. It may be possible to establish a relationship between radius and generation number, counting from below. Thus

$$\mathbf{r}_{\mathbf{u}} = \mathbf{r}_{\mathbf{0}} \mathbf{K}_{\mathbf{1}}^{\mathbf{u}}$$

may be applicable to an asymmetrical system, the implication being that branches with similar quantitative function are grouped together.

#### Path Lengths

In a symmetrical system path lengths from the stem branch to each end branch are identical, but in the asym-



Generations down (w)

Generations up (u)

FIG. 2. Two methods of numbering the generations in an asymmetrical dichotomously branching system. The branches may be numbered downward starting with the main stem as zero; this method is similar to that used in the symmetrical system. Alternatively, the branches may be numbered upward starting with the end branches as zero.



FIG. 3. Derivation of an asymmetrical system from a symmetrical system. A: a symmetrical system numbered upward; B: removal of part of the system to produce C; C: the resulting asymmetrical system is correctly numbered upward without changing the numbering of any branch, similarly sized branches having the same number.



metrical system of Fig. 2 the path lengths will vary according to the route chosen, and this variation will be a measure of the asymmetry. If the length of each branch were known, then the values for all the possible path lengths could be computed.

Similarly, the mean time which it would take a molecule of gas to traverse a single branch under defined flow conditions could be calculated. Thus, the time taken by molecules of gas to travel from the carina to an end branch (the transit time) could be found by summation of the individual values.

#### METHODS AND MATERIALS

Since a resin cast of the bronchial tree was to be used for making the measurements, a pilot study was first carried out to determine the most suitable methods. The results suggested that a detailed study of one pair of lungs would yield more information than an incomplete study of several. It was required that the lungs studied should be normal. This proved a difficult requirement to satisfy, but eventually the lungs were taken from a 25year-old male who had died from renal failure complicating diabetes mellitus. There had been no history of lung disease and clinically no terminal pneumonia had occurred.

The lungs were removed intact at autopsy together with the trachea and major bronchi. Their gross appearance was normal except for three patches of consolidation about 1 cm in diameter at the costophrenic angles. Little secretion was seen in the large airways.

The lungs were inflated and fixed by the method previously described (9) in which the lungs are inflated FIG. 4. Cast of an acinus incompletely filled with resin. The smooth terminal bronchiole (seen at the lowest margin of the illustration) divides into two respiratory bronchioles, the upper one of which shows alveolar markings. After two or three generations the respiratory bronchioles divide to give rise to alveolar ducts which are completely surrounded by alveoli. The rounded protuberances which are produced by partial filling of the alveoli with the resin are seen on the peripheral parts of the cast.

to a volume of 5 liters corresponding to the position of inspiration in life and fixed using 4% formalin solution. The resin cast was then prepared using a modification of the method described by Tompsett (15). The resin used has the advantage of not shrinking on setting.

On inspection of the finished cast it was seen to be complete down to terminal bronchiolar level, with only a few broken small branches, and with incomplete filling of the three areas of consolidation. The airways distal to the terminal bronchioles were filled with resin in variable degree but never completely, so that the respiratory bronchioles and proximal alveolar ducts could be made out in most cases.

Such a partly filled acinus is shown in Fig. 4. The terminal bronchiole may be identified by the absence of the smooth protruberances which indicate the openings of alveoli and which result from the polymerization of the resin just as it is about to flow into an alveolus. In a few areas resin had progressed further, when the large quantity of resin present made identification difficult, but indicated that flow into the alveolus proper could be achieved.

The terminal bronchiole gives rise to three generations of respiratory bronchioles (on average). These structures bear the impressions of alveolar openings on their walls and the number of such openings increases distally, until in the third generation more than half of the wall is occupied by the openings. A side devoid of openings is often found where the adjacent branch of the pulmonary artery runs.

Finally, there are several generations of alveolar ducts, distinguished by the fact that their walls consist entirely of the rounded protuberances marking alveolar openings.



FIG. 5. Measurement of bronchial dimensions. A: shows how length was measured between the two points of bifurcation, a and b. B: measurement of a tapering branch at the midpoint of its diameter. C: measurement of the maximum and minimum diameters of a branch elliptical in section.

## Measuring Completed Cast

It was proposed to measure the length and diameter of every structure in the lung cast to an arbitrary anatomical site, defined as that point where the measured diameter first became equal to or less than 0.07 cm (700  $\mu$ ). The large number of branches distal to this could then be measured by a sampling technique.

The segment of lung distal to the first branch of diameter 0.07 cm or less was designated a "lobule" and the branch supplying it a "lobular branch." It should be noted that this designation is purely arbitrary and implies no a priori functional significance. All the lobular branches of the cast were then broken off and a number of these branches from each anatomical bronchopulmonary segment were reserved for study.

The whole cast, pruned down to lobular level, was next divided into bronchopulmonary segments to make the work of measuring easier and a semidiagrammatic drawing was made of every segment.

The length of each branch was measured between the two points of bifurcation as indicated in Fig. 5A. The best method of making this measurement posed a problem, and the technique finally used was to apply the needle points of a pair of screw-controlled dividers to points a and b. The distance between the points was then measured with a low-power traveling microscope having a calibrated grid eyepiece. Distances greater than its compass were measured using the vernier scale on the microscope traverse. Diameter was measured in a similar way, tapering branches were measured at the midpoint of their length, and those with elliptic sections were measured in both major and minor axes, using the arithmetic mean value (see Fig. 5B and C). Length was measured to the nearest 0.02 cm and diameter to 0.01 cm.

The identity and location of each structure within the cast was designated by three numbers. The first was a unique serial number, the second showed its position counting down from the trachea (designated zero), and the third showed it counting up from the first branch of 0.07 cm or more (designated 1). Thus, lobular branches of 0.07 cm are designated 1 and those 0.04–0.06 cm diameter are designated zero.

The location, identity, diameter, and length of each structure was thus reduced to numerical terms, and these were transferred to IBM punched cards, one card being used for each structure. TABLE 1. Mean error variance, standard deviation of duplicate readings, and coefficient of variation of 30 duplicated measurements of length and diameter

	Length, mm	Diameter, mm
Mean error variance Standard deviation	0.0067 0.08	0.0015 0.04
Coefficient of variation, $\%$	2.1	3.1

# Sampling Technique for Smaller Structures

From each bronchopulmonary segment approximately 20 lobular branches were taken for study. The ideal method of selection by random numbers was impossible since the lobular branches in the middle of the segment could not be seen until the peripheral branches had been pruned off. The alternative plan of a stratified sampling procedure was adopted; structures arising from branches between 0.4 and 0.7 mm in diameter were taken from different regions of each bronchopulmonary segment.

Having selected 313 lobules, each one was examined under a binocular dissecting microscope, recording the pattern of branching as before, observations being made down to the last order of respiratory bronchiole—viz., that one giving rise to an alveolar duct. For the purpose of this study, therefore, the last respiratory bronchiole was regarded as an end branch and the value for E, which indicates a number of structures, refers to the number of distal respiratory bronchioles supplied by any branch.

The number of divisions from the lobular branch to the distal respiratory bronchiole was counted along 8,536 pathways and the number of distal respiratory bronchioles in each lobule was counted.

Two lobules in each of fourteen bronchopulmonary segments were reserved for detailed study. Within these lobules the length and diameter of every structure down to the terminal respiratory bronchiole was measured, a total of 452 intralobular pathways.

In order to assess the precision of measurement they were repeated on a sample of 30 branches on different occasions. The mean error variance, standard deviation of duplicate readings, and the coefficient of variation are in given in Table 1.

# Calculations

An IBM 7070 computer was used for computation and the cards containing the data were prepared as described. The program first computed the path lengths of all possible pathways from the carina to the lobular branches ( $\Sigma$ L). This value for  $\Sigma$ L enabled a value of E to be computed for all the lobular branches, using the regression equation given in the APPENDIX. All the values of E were then added from the lobular branches to the carina, the value for any branch being the sum of the values for its daughter branches. Where a branch was broken, its value of E was zero. This error was corrected by determining the mean value of E for all branches of similar diameter within the same bronchopulmonary segment and adding this to the broken branch.

From the values of length and diameter and the corrected values of E for each branch, several derivatives were calculated; viz., transit time, pressure gradients, and total volume.

### Computation of Transit Time and its Distribution

The mean time taken by a molecule of gas to traverse a cylindrical tube in which gas is flowing is the transit time for that cylinder. Similarly, the time taken for a molecule to traverse any pathway, however complex, can be computed by summing the transit times of the components of the pathway, which may be expressed as simple cylinders.

Since measurements are all available from the lung cast, all the possible transit times can be computed but, in order to do this, two assumptions are necessary: I) that the volume of lung supplied by the distal respiratory bronchiole is identical throughout the lung and 2) that the increase in volume induced by the fall in intrathoracic pressure is the same everywhere and that the increase in volume is everywhere in phase.

Both of these assumptions are untrue. First, there must be a range of volumes supplied by each distal respiratory bronchiole. In so far as these are randomly distributed and the range is small there will not be a big error. Second, the effects of regional variations in lung expansion due to gravity, turbulent flow, variation in bronchial dimensions due to muscle tone, lung volume changes, and blurring of the square-wave front are all excluded. Thus, the calculations which follow relate purely to the effects of bronchial anatomy and laminar flow.

Given these assumptions the flow rate of gas in any given branch will be directly proportional to the number of distal respiratory bronchioles arising from that branch.

 $\dot{\mathbf{V}} \propto \mathbf{E}$ 

Since the transit time in a right cylinder is its volume (V) divided by the flow rate  $(\dot{V})$  we may now write

Transit time 
$$\propto \frac{V}{E}$$

Since both E and V are definable, TT is readily expressed in arbitrary units, which may be converted directly into seconds for any given value of tracheal flow rate.

# Distribution of Flow and Resistance in Airways

The resistance R to laminar flow in a cylinder brought about by a pressure gradient may be expressed by Poiseuille's equation and the resistance to flow is also given by the pressure gradient divided by the flow rate. Thus

$$R = \frac{8\eta}{\pi r}$$

and

$$R = \frac{r_1 - r_2}{\dot{V}}$$
$$\therefore \frac{P_1 - P_2}{\dot{V}} = \frac{8\eta I}{\dot{V}}$$

It can be derived (see APPENDIX) that the pressure gradient may be expressed in anatomical terms as

$$P_1 - P_2 \propto \frac{LE}{D^4}$$

where D is the diameter of the tubes of length L.

This pressure gradient may be summed along any pathway, since the resistances producing it are in series, so that the total pressure gradient  $\Sigma(P_1 - P_2)$  may be found as for  $\Sigma L$ .

The values of the summed transit times and summed  $(P_1 - P_2)$  were calculated for all possible routes from the carina to the lobular branches. In the computation of transit times the total volume of the airways down to lobular branches was also found.

## RESULTS

Between the trachea and branches of 0.07 cm diameter a total of 8,298 branches were measured, of which 1,603 were lobular branches of 0.07 cm diameter. In addition, 4,614 lobular branches of 0.04–0.06 cm diameter were measured with respect to diameter only. Two hundred fifty-two branches were broken and these gave rise to an estimated 10% of the total number of distal respiratory bronchioles.

The total number of distal respiratory bronchioles supplied by the trachea was 233,941.

#### Total Volume

From the carina to the lobular branches the airways had a computed volume of 71 ml. In order to compare this measurement with other published data it is necessary to estimate the volume of the upper airways from carina to the mouth. Published estimates of this volume vary, Landahl (10) gave a figure of 65 ml while Rohrer (12) suggested 90 ml. Comroe et al. (3) states that a tracheostomy bypasses 50 ml of dead space, while Bates and Christie (1) give 100 ml as the bypassed volume. Bouhuys (2) quotes 71 ml as the extrathoracic dead



FIG. 6. Frequency distribution of the number of divisions down to the lobular branches.



FIG. 7. Measured number of branches  $(N_u)$  in each generation (u) up the bronchial tree.  $N_u$  is plotted on a logarithmic scale. The regression line has the equation  $N_u = 1.38^{25-u}$  which should be compared with that for a symmetrical system of 25 generations which is  $N_u = 2^{25-u}$ .



FIG. 8. Frequency distribution of path length ( $\Sigma$ L) to lobular branches. There is a suggestion of a bimodal distribution, possibly due to the difference in length between the right and left main bronchi.

space. If a mean value of 80 ml for the volume of the upper airways is assumed, this gives a volume of 150 ml down to the lobular branches. This is the mean value for the anatomical dead space in males, measured by the single-breath technique and suggests that the anatomical dead space as measured by physiological techniques corresponds to the volume down to the region of the lobular branches. It should be recognized that this is a crude projection of the data and it is not desired to attach any great significance to the correspondence of the two volumes.

# Measurements Down to Lobular Branches

The lobular branch with the shortest path length was reached after 8 dichotomous branchings and the longest path length was found after 25 such branchings. The mode of the distribution of divisions down is 14 and the mean is 14.6. This distribution is shown in Fig. 6.

Figure 7 shows the logarithm of the number of branches in each generation, counting from below up. The linear relationship shows its similarity to a symmetrical system. The distribution of path lengths and transit times is shown in Figs. 8 and 9, respectively.

# Measurements of Intralobular Branches

Since it was possible that the detailed structure of the airways distal to the lobular branch differed from lobule

to lobule and, perhaps, from one region of the lung to another, a sample of the 313 lobules was measured as described earlier.

The mean number of divisions in the 313 lobules has been plotted against the diameter of the supplying branch in Fig. 10 and it will be seen that the relationship is almost linear. The range was from 2 to 7 divisions.

Since it was possible that the distribution of lobular branch sizes differed as between short and long path lengths, a test for this was made. The results indicated that the distribution of lobular branch sizes was independent of path length.

# Path Lengths and Transit Times Within Lobules

The distribution of path lengths is shown in Fig. 11, the range being from 2 to 9 mm. The distribution of transit times is shown in Fig. 12, the range being from 75 to 325 units.

Because the intralobular values of  $\Sigma L$  and  $\Sigma TT$  showed little regional variation, it was throught valid to pool the data from all the 313 lobules studied.



FIG. 9. Frequency distribution of transit time ( $\Sigma$ TT) to lobular branches.  $\Sigma$ TT is expressed in arbitrary units.



FIG. 10. Mean number of divisions from the lobular branches to the distal respiratory bronchioles. The straight line is drawn through the three higher points.



FIG. 11. Frequency distribution of path length ( $\Sigma L$ ) from the lobular branches to the distal respiratory bronchioles.



FIG. 12. Frequency distribution of transit time ( $\Sigma$ TT) from the lobular branches to the distal respiratory bronchioles.  $\Sigma$ TT is in arbitrary units.



FIG. 13. Frequency distribution of transit time ( $\Sigma$ TT) from the carina to the distal respiratory bronchioles (the beginning of the first alveolar ducts).  $\Sigma$ TT is expressed in arbitrary units.



FIG. 14. This figure shows the mean length (L) of branches of a given diameter (D). Diameters have been rounded to the nearest 0.1 mm, with a maximum value of 4.0 mm. The equation for the regression line is L = 1.10 + 2.57 D.



FIG. 15. Same information as Fig. 14, but for branches of diameter 4–16 mm. The diameters have been grouped at intervals of 1.0 mm.

#### Combined Data from All Measurements

Path lengths. The distribution of  $\Sigma L$  down to lobular branches ranges from 7.5 to 21.5 cm. Within the lobules the path lengths are only short, ranging from 2 to 9 mm, which would give a total range from the carina to the distal respiratory bronchioles of 7.7-22.4 cm. In round figures, the range is from 8 to 22 cm.

Transit times. In this case, the branches within the lobules contribute significantly to the total, the range being 75-325 units. The distribution for lobules may be added to that for the larger branches to give a total distribution by the method of convolution. The combined frequency distribution is shown in Fig. 13.



FIG. 16. Relationship between the diameter (D) of bronchi and their generation number (u) counting from below. The ordinate expresses the mean logarithm of the diameters of all the branches in a given generation. The data may be (arbitrarily) divided into three groups, each with a regression equation which may be written  $D_u = a \times b^u$ . The constants are as follows

a	b	
0.676	1.175	
1.340	1.062	
0.330	1.161	
	a 0.676 1.340 0.330	

D in units of 1.0 mm.



FIG. 17. The ordinate expresses the mean value of the logarithm of the number of distal respiratory bronchioles (E) supplied by branches whose diameter (D) is represented on the abscissa. The abscissa shows the logarithm of the diameters rounded to the nearest 0.1 mm. The data may be divided (arbitrarily) into three groups, each with a regression equation which may be written  $E_D = a D^{\circ}$ . The constants are as follows

D, mm	Log D	a	с
0.4-1.6	1.40-0.20	91.2	1.96
1.7-5.0	0.23-0.70	14.4	3.84
5.1-16.0	0.72-1.20	186.6	2.24



FIG. 18. Mean value of the logarithm of the number of distal respiratory bronchioles (E) supplied by branches of each generation (u) counting upward. The regression line has the equation  $E_u = 45.7 \times 1.39^u$ . It has been extrapolated as the dotted line which indicates, where it intersects the abscissa, that 10 divisions would be expected between a lobular branch and a distal respiratory bronchiole. The fact that only 5 or 6 are found indicates a change in the pattern of branching in the terminal airways toward symmetrical dichotomy.

Divisions down. The number of divisions down to lobular branches ranges from 8 to 25, while the number of divisions within lobules ranges from 2 to 7. This gives a total range of 10-32 divisions from carina to the distal respiratory bronchioles.

# Interrelationships Between Measurements

The most convenient way of giving this data is in the form of figures and equations. These are shown in Figs. 14–18.

### Asymmetrical Nature of Normal Airways

The whole of the data obtained from the measurements on the lung cast are summarized in Table 2 which illustrates the asymmetrical structure of the airways and how it may be dealt with by counting from below up. Table 2 may be compared with a similar one published by Weibel (16, p. 139) based on the assumption of a symmetrical structure.

# Pattern of Branching in Bronchial Tree

The transport of gas down the bronchial tree and its transfer across the alveolar capillary membrane have conflicting requirements. Thus, to minimize resistance to flow the volume of the conducting airways should be maximized. This, however, encroaches upon the volume required for gas transfer so that some compromise between these aims is necessary. This has been achieved by an appropriate relation between the angles of branching and the dimensions of bronchi at different levels.

A theoretical analysis of this situation has been reported elsewhere (8) and indicates the excellence with which lung structure has been related to the functional requirements. Branching in the bronchial tree is by asymmetrical dichotomy and no evidence was found for monopodial branching. The occasional trichotomy might be explained by shortening of a single dichotomous branch.

Within the lobules branching is more nearly symmetrical, as reflected in the number of branches predicted and actually found. Taking the 5,000 lobular branches (approximately) five symmetrical dichotomies would result in 160,000 distal respiratory bronchioles. In fact the measurements here reported indicate some 224,000 such structures, suggesting between 5 and 6 dichotomous divisions. These results show the more symmetrical arrangement of the intralobular structures.

A notable feature was the arrangement of the bronchial supply to alveoli situated near the hilum of the lung. In all cases these were supplied by recurrent bronchi which turned back from the larger intrasegmental

TABLE 2.	Asymmetrical	model	of	the	airways	derived	from
measureme	nt of cast						

Structure	Genera- tion Up	No.	Diame- ter, mm	Length, mm	Value of E
Trachea	25	1	16.0	100	223 041
Tracinca	23	1	12.0	40	123,541
	23	2	10.3	26	88 920
	20	2	8.9	18	63 073
	21	2	7 7	14	46 026
	20	3	6.6	11	33 113
	19	6	57	10	23 823
	18	8	49	10	17 140
	17	12	4 2	10	12 331
	16	14	3 5	10	8 872
	15	20	33	96	6 383
	14	30	3.1	9.1	4.592
	13	37	2.9	8.6	3,304
	12	46	2.8	8.2	2.737
	11	64	2.6	78	1 710
	10	85	2.4	7.4	1.230
	9	114	2.3	7.0	885
	8	158	2.2	6.7	637
	7	221	2.0	6.3	458
	6	341	1.78	5.7	330
	5	499	1.51	5.0	237
	4	760	1.29	4.4	171
	3	1,104	1.10	3.9	123
	2	1,675	0.93	3.5	88
	1	2,843	0.79	3.1	64
Terminal bron- chiole	(-2)	27,992	0.60		8
Distal respiratory bronchiole	(-5)	223,941	0.40		1

u: minus values for the terminal bronchiole and distal respiratory bronchiole are included to give an approximate indication of the number of divisions between the structures in a lobule.  $N_u$ : values of N are those actually observed for each value of u.  $D_u$ : diameters are those computed from the three regression equations relating  $D_u$  and u.  $L_D$ : lengths are computed from the regression equation relating D and L up to D = 4.0. Above this they are taken from the line of fit, drawn by eye, on the graph in Fig. 15.  $E_u$ : values of E for  $u \ge 1$  are those computed from the regression equation relating  $E_u$  and u. Trachea: dimensions given are those actually measured.



FIG. 19. Mode of supply of respiratory units near the main bronchi by recurrent branches. Branches taking the shorter route direct from the main bronchi are not found.

branches through  $180^{\circ}$  or more, as shown in Fig. 19. This has the effect of increasing  $\Sigma L$ ,  $\Sigma TT$ , and  $\Sigma (P_1 - P_2)$  to the most proximal alveoli, thereby reducing the total range of these values for the whole lung.

#### DISCUSSION

It is of interest to compare some of these findings with those of other authors. The only previous worker to study the asymmetry of the bronchial tree was Ross (13). He worked with dog's lungs and since there is a species difference in bronchial anatomy, direct comparison is difficult. However, he did find that the number of terminal branches arising from a bronchus of any given diameter decreased with increasing path length. Our data show a similar relationship although the methods of mathematical analysis were different (see equation 13 in the APPENDIX and Fig. 20).

In Table 3 the number of branches 1.5 mm in diameter and the number of terminal bronchioles as given by three previous authors are compared with our data. There is fairly close agreement for branches of 1.5 mm diameter, but Weibel's (16) estimate of  $66 \times 10^3$ terminal bronchioles is approximately double ours of  $28 \times 10^3$ . At first glance this appears to be a large discrepancy but, in fact, it represents only one generation of dichotomous branching. Thus, Weibel gives generation 16 as representing the terminal bronchioles, whereas if he had given generation 15, the number would have been  $33 \times 10^3$ , an estimate quite close to ours. Since Weibel's method of determining the generation of structures in the distal part of the bronchial tree depends explicitly on the assumption of symmetrical dichotomous branching right down to the distal alveolar ducts, it is perhaps remarkable that the estimates are so close.

Again, comparison of the estimates of volume of the bronchial tree is difficult owing to the impossibility of selecting similar anatomical points in the respective models. However, taking the mean number of division down to lobular branches as 14 and comparing with Weibel's generation 13 (assuming they are 1 out phase as above) the volume from the carina is given by Weibel as 75.5 ml compared with our estimate of 71 ml. In general, then, our results are comparable to those of other workers and such differences as do arise are probably due to the method of analysis which has been used to deal with the asymmetry of the bronchial tree.

With respect to the numbering of generations of branches all previous workers have counted down from the trachea. This method is unsuitable for asymmetrical systems. By counting the generations upward from branches of 0.07 cm diameter to the trachea, a close relationship was demonstrated between the diameter of a branch, the number of distal respiratory bronchioles it supplies, and its generation number.

#### APPENDIX

List of Abbreviations

A = cross-sectional area of a branch

- D = diameter of a branch
- E = number of end branches (distal respiratory bronchioles) arising from a given branch
- L = length of a branch
- N = number of branches in a given generation
- $P_1 P_2$  = pressure gradient between the ends of a tube or branch
  - R = resistance to flow
    - $\mathbf{r} = \mathbf{radius} \mathbf{of} \mathbf{a} \mathbf{branch}$
  - TT = transit time
  - U = total number of divisions counting upward from an end branch to the main stem
  - u = number of divisions up to a given branch from an end branch
  - V = volume
  - $\dot{\mathbf{V}} = \mathbf{flow} \ \mathbf{rate}$
  - w = number of divisions down from the main stem to a given branch
  - $\Sigma L$  = summed lengths of the branches along a given pathway
- $\Sigma(P_1 P_2)$  = summed pressure differences of the branches along a given pathway
  - $\Sigma TT =$  summed transit times of the branches along a given pathway

Suffixes are used as shown in the following examples

- $D_u$  = diameter of a branch u divisions up
- $E_D$  = number of end branches supplied by a branch of diameter D
- $r_w$  = radius of a branch w divisions down



FIG. 20. The abscissa shows the path length ( $\Sigma$ L) from the carina to a lobular branch. The ordinate represents the number of terminal respiratory bronchioles (E) supplied per square millimeter cross-sectional area (A) of the lobular branch. Data are from a sample of 105 lobules.

table 3.	Comparison	of the	numbers	of bronchi
as estimat	ed by differen	at authoria	ors	

Bronchus	Bronchus Rohrer*		Findeisen Weibel	
1.5 mm diam- eter			512	781†
Terminal bronchiole	$30 \times 10^3$	54 × 103	66 × 10 <sup>3</sup>	28 × 10 <sup>3</sup>

\* Rohrer's data for one lung has been multiplied by 2. † These include branches 1.4–1.6 mm diameter.

#### Radius of a Branch in a Symmetrical System

If there is a constant proportional decrease in the radius r of branches in each successive generation down the system, then

$$\frac{\mathbf{r}_{\mathbf{w}}}{\mathbf{r}_{\mathbf{w}-1}} = \mathbf{K} \tag{1}$$

Where K is a constant less than unity,  $r_w$  is the radius of a branch in generation w, and  $r_{w-1}$  is the radius of a branch in the preceding generation.

$$\mathbf{r}_{\mathbf{w}} = \mathbf{K}\mathbf{r}_{\mathbf{w}-1} \tag{2}$$

$$= K^2 r_{w-2} \tag{3}$$

$$= K^{w}r_{w-w}$$
(4)

(5)

 $\mathbf{r}_{\mathbf{w}} = \mathbf{K}^{\mathbf{w}}\mathbf{r}_{0}$ 

where  $r_0$  is the radius of the main stem.

Similarly counting up the symmetrical system it can be shown that

$$\mathbf{r}_{\mathbf{u}} = \mathbf{K}_1^{\mathbf{u}} \mathbf{r}_0 \tag{6}$$

Where  $K_1$  is a constant greater than unity,  $r_0$  is the radius of an end branch, and  $r_u$  is the radius of a branch u divisions up.

#### Calculation of the Pressure Gradient

Considering a cylindrical tube along which there is a laminar flow of gas, then the resistance R is defined by

$$\mathbf{R} = \frac{\mathbf{P}_1 - \mathbf{P}_2}{\mathbf{\dot{V}}} \tag{7}$$

where  $P_1 - P_2$  is the pressure gradient along the tube and  $\dot{V}$  is the flow rate.

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Equation 7 may be expressed as

$$\mathbf{P}_1 - \mathbf{P}_2 = \mathbf{R}\mathbf{V} \tag{8}$$

From Poiseuille's equation

$$R = \frac{8\eta L}{\pi r^4} \tag{9}$$

where  $\eta$  is the viscosity of the gas and L and r are the length and radius of the tube, respectively.

From equation 9

$$R \propto \frac{L}{r^4}$$
 (10)

It has been shown previously that given certain assumptions

$$\dot{\mathbf{V}} \propto \mathbf{E}$$
 (11)

By substituting for R and  $\dot{V}$  from equations 10 and 11 into 8 we get

$$\mathbf{P}_1 - \mathbf{P}_2 \propto \frac{\mathbf{LE}}{\mathbf{r}^4} \tag{12}$$

(The numerical values for  $P_1 - P_2$  were computed in practice substituting  $16/D^4$  for  $1/r^4$ .)

#### Regression Equation for Value of E

The value of E for a lobular branch was found to be proportional to the cross-sectional area A of the branch. The value E/A for a randomly chosen one-third sample of all the lobules studied was plotted against  $\Sigma L$  to the lobule (Fig. 20). A significant correlation was found (0.05 > P > 0.02) although the correlation was low (r = -0.202).

The best estimate of E/A from  $\Sigma L$  (in units of 1 mm) is given by

$$E/A = 151.6 - 0.244\Sigma L$$
 (13)

$$E = A(151.6 - 0.244\Sigma L)$$
 (14)

$$E = \frac{D^2 \pi}{4} (151.6 - 0.244 \Sigma L)$$
 (15)

Equation 15 was used to compute the value of E for every lobular branch after  $\Sigma L$  to the branch had been calculated.

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